ISSN 2229-5518

Temporal Evolution of Viscous Fingering in Hele Shaw Cell: A Fractal Approach

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Abstract—Viscous fingering in radial Hele Shaw cell is studied and the temporal evolution of patterns in terms of the structural complexity of shape is presented using concept of fractal and fractal geometry. HP90, a viscous servo gear oil is used as viscous medium and the low viscosity medium is air. Air is suddenly injected between the two plates of the Hele Shaw cell and fingering patterns are recorded using video camera to study time course of evolu-tion of the fingering patterns. From the video recording Frames at suitable time interval are selected and are separat-ed for analysis. The images so obtained are converted into gray scale and using appropriate threshold they are con-verted into two colour bitmaps. Box counting is implemented on the two colour bitmap images for determining frac-tal dimensions at various stages. It is found that the complexity of shape and structure remains more or less the same as is revealed from the associated fractal dimensions, details are presented.

Key words— Fractal, Hele Shaw cell, Viscous Fingering, dendritic, Box counting, self similarity, scale invariance.

1 INTRODUCTION

Fractal patterns characterize the non-equilibrium growth, such as the dendritic shape of snowflakes [1], the shape of bacteria colonies growing in stressed environments [2], electro-deposition [3], dielectric breakdown [4] and viscous fingering etc. [5]. All these growth processes lead to structures that are complex in shape and the parameters can be charac-terized on the basis of Fractal Dimension using fractal geome-try as traditional Euclidian geometry can not be used.

When a less viscous fluid is forced into a more viscous fluid under pressure, the interface between the two becomes unsta-ble and the less viscous fluid enter by force into the more vis-cous 'defending' fluid which leads to the formation of finger like pattern known as 'viscous fingers' [6] which may repeat-edly branch and sometimes form a fractal pattern. Under ap-propriate approximations Laplace Equation can express the interface, as in diffusion-limited processes

$$\nabla^2 u(x, t) = 0 \quad \ldots\ldots.. \quad (1)$$

with suitable boundary conditions.

Viscous fingering was first observed by petroleum engi-neers when an aqueous solution displaces more viscous oil in underground reservoirs. During secondary oil recovery [7], the study of viscous fingering the phenomenon is important in problems such as fluid flow in porous media [8,9], dendritic solidification, combustion in two dimensions [10] and electro-chemical deposition [11], Lot of research effort have been tak-en by workers in the field to understand the various physical factors governing Viscous fingering.

2. VISCOUS FINGERING IN HELE SHAW CELL.

Viscous fingering in radial Hele Shaw cell is studied using high viscosity gear oil HP90, the construction of the radial Hele Shaw cell is discussed elsewhere [12]. We used thick oils with high viscosity as high viscosity fluid between the two plates of the Hale Shaw cell and air as low viscosity fluid. Air is used as he low viscous fluid to displace the more viscous fluid. The cell was carefully leveled and space between the two plates adjusted and was filled with the more viscous fluid. Spacers were used between the cell plates to ensure proper and uniform spacing. The two plates were firmly clamped to avoid any displacement of the plates due to sudden change in pressure arising from the rapid injection of less viscous fluid, which is air under pressure.

To record the temporal growth the cell was illuminated with diffused light for photography and video recording. As the formation of fingering patterns is and instantaneous process and is over within second depending on the working conditions, 'still' photography becomes difficult. Fast framing video recording provides finer detail of evolution revealing temporal progress of the fingering patterns.

The fingering pattern obtained using HP90 oil as a more viscous fluid and air as less viscous fluid under rapid injection are as shown in Figs. 1. The fingering patterns exhibits seven prominent primary branches with secondary branches. This is the last frame used for the study of temporal growth of the fingering pattern. The evolution of the viscous fingering pattern was recorded using a video camera at a framing rate of 37 frames per second. This corresponds to an interval of 0.027027

ISSN 2229-5518

seconds between two consecutive frames recorded in the video which was found to be too short compared to the growth process.

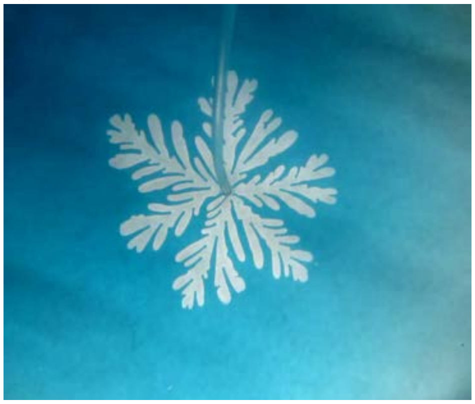
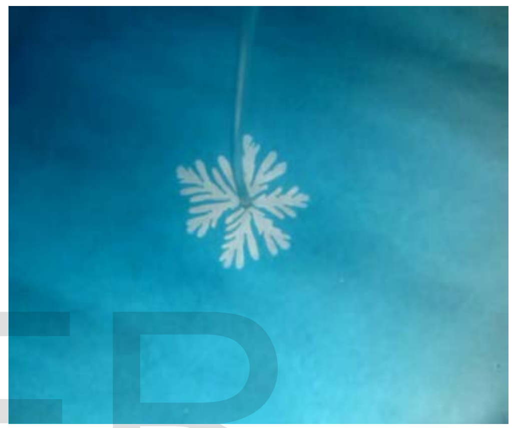


Fig. 1. Fully developed viscous fingering pattern obtained in radial Hale Shaw Cell using High viscosity oil HP90.

The entire development of the fingering pattern took little less than 4 second occupying more than 120 frames, in view of this selected stages during the development were used at regular intervals and the last frames where the fingering pattern tend to slow down due to fall of pressure were discarded. Since the process lasted for more than two seconds, we selected frames with a difference of 15 which corresponds to a time interval of about 0.4 seconds, however the first stage was selected at the 20th frame rather than 15th to accommodate appreciable growth. In all there are 8 stages including the first one at time = 0 where there is no progress and the last one shown in Fig. 1. Leaving these two frames rest of the six stage frames are shown in Fig. 2.



ISSN 2229-5518

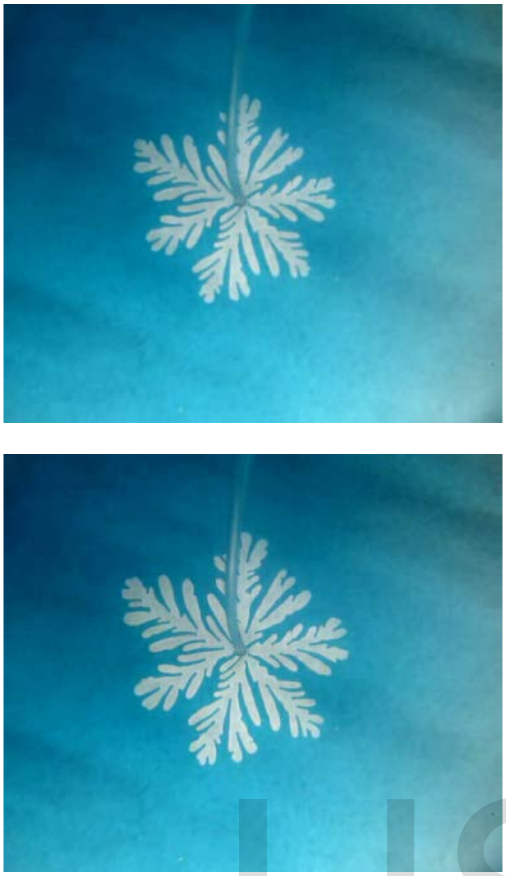
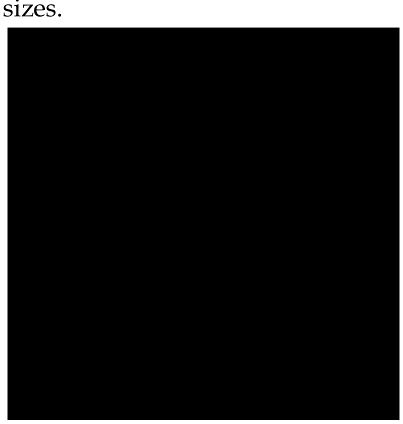
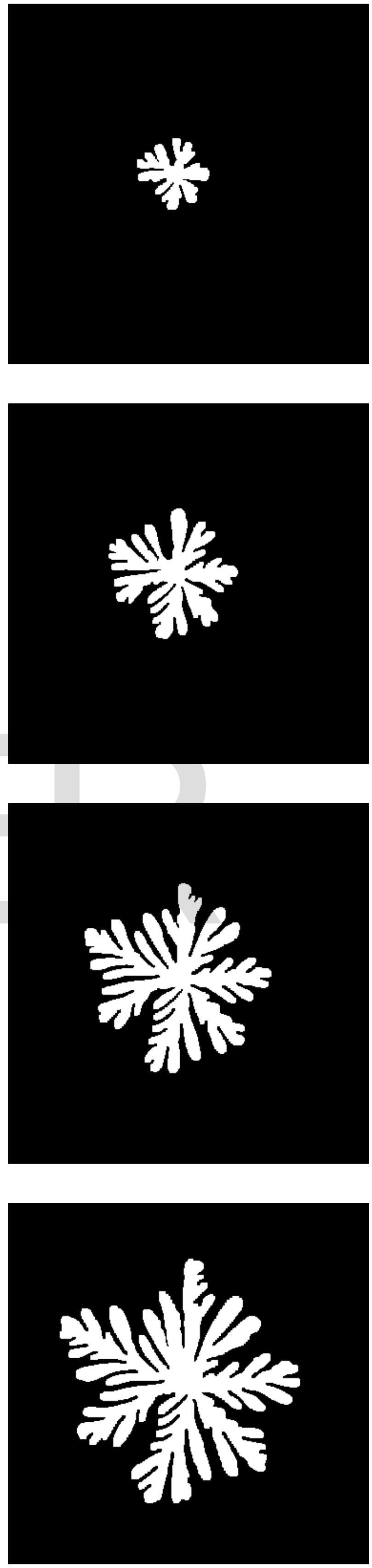


Fig. 2. Six stage of growth of fingering pattern, first stage being blank is not shown and the last stage is shown in Fig. 1.

The images so obtained from the separation and selection of the frames shown in Fig. 1 and 2 were processed, converted to gray scale and finally converted to two colour bitmap images using appropriate threshold and are shown in Fig. 3. These two colour bitmap images were then analyzed using computer program for implementation of the box-counting using boxes of different sizes.





ISSN 2229-5518

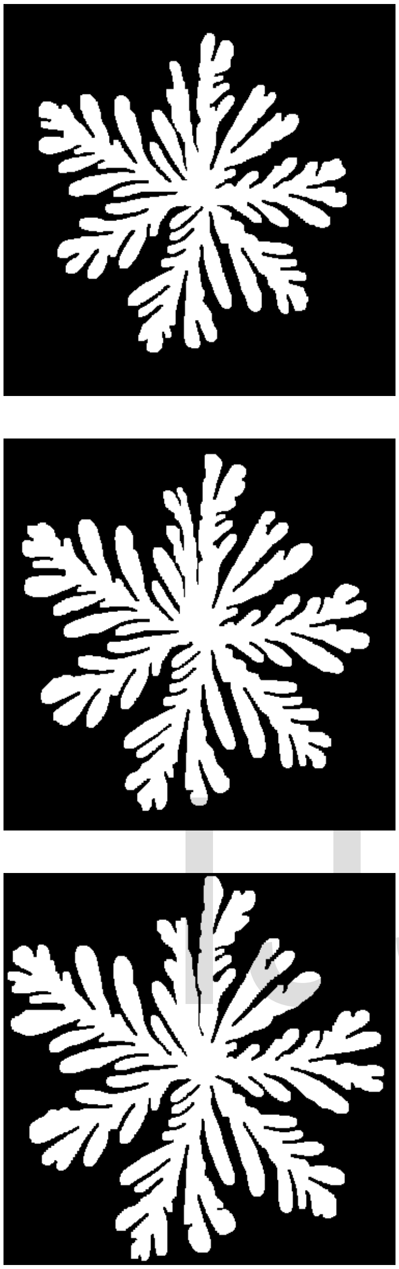


Fig. 3. All stage of growth of fingering pattern including first stage (blank) and the last stage shown in Fig. 1.

The results of box counting show that all these patterns have self similarity and scale invariance and thus the shapes are fractals in nature obeying scaling law. The results of box counting are presented in fig. 4 – 10 in the form of plot of log(N) versus log(r).

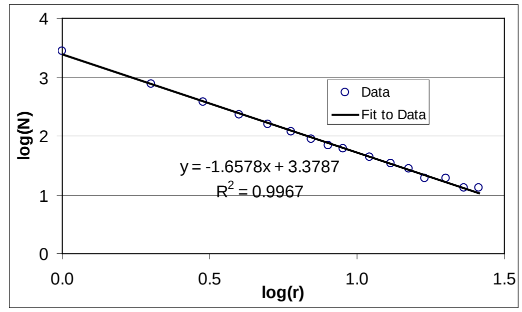


Fig. 4. log(N) versus log(r) plot for stage – 1, second image of Fig. 3.

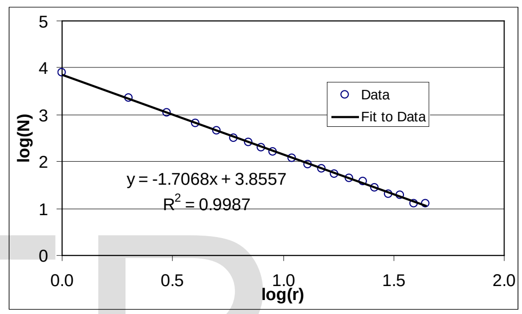


Fig. 5. log(N) versus log(r) plot for stage – 2, third image of Fig. 3.

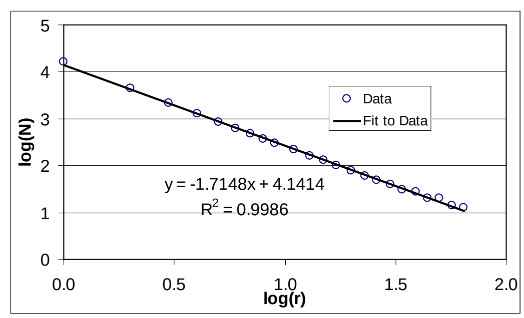


Fig. 6. log(N) versus log(r) plot for stage – 3, fourth image of Fig. 3.

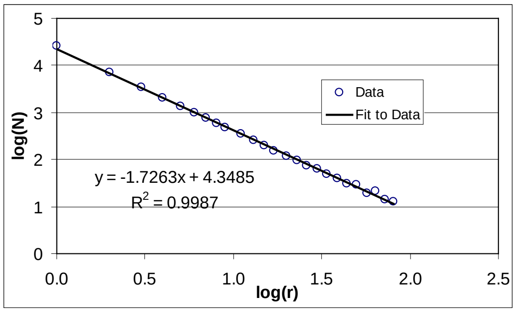


Fig. 7. log(N) versus log(r) plot for stage – 4, fifth image of Fig. 3.

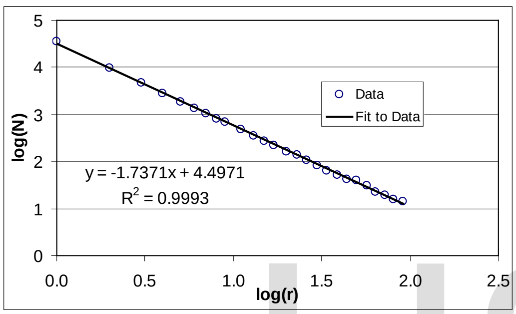


Fig. 8. log(N) versus log(r) plot for stage – 5, sixth image of Fig. 3.

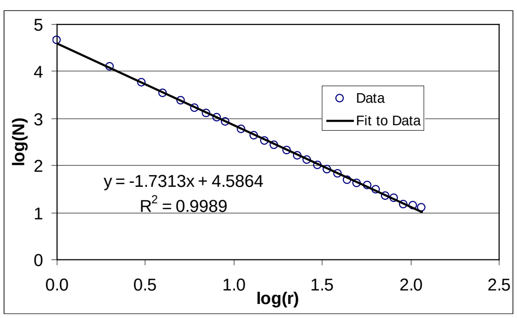


Fig. 9. log(N) versus log(r) plot for stage – 6, seventh image of Fig. 3.

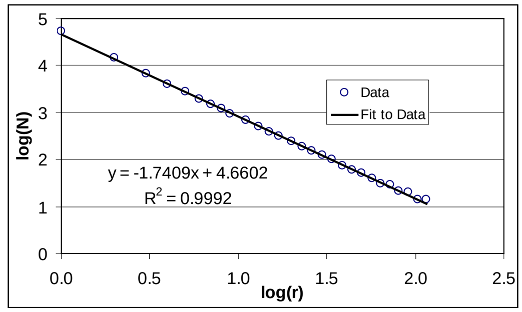


Fig. 10. log(N) versus log(r) plot for stage – 7, eighth image of Fig. 3.

For all the plots shown in Fig. 4 – 10, the points plotted are the actual data resulting from the implementation of box counting and the straight line joining the points is the best fitting (least square fit) straight line to the points. The equation shown in the inset is the equation of the fitting straight line and the coefficient of x is the slope of the straight line. All the points lie well along a straight line in all the plots under discussion indicating the power law holds and the patterns exhibit fractal characters obeying scale invariance and self similarity. The fractal dimensions of the seven patterns at seven different stages of the growth of fingering process obtained from the slope of straight line corresponding to the respective stage are shown in Table – 1.

TABLE – 1
FRACTAL DIMENSION AT DIFFERENT STAGES OF FINGERING PATTERN

Stage	Time	FD
1	0.5405	1.6578
2	0.9459	1.7068
3	1.3514	1.7148
4	1.7568	1.7263
5	2.1622	1.7371
6	2.5676	1.7313
7	2.9730	1.7409

It is seen from the plots and fractal dimension shown in Table – 1 that the complexity of shape associated with the fingering pattern remains more or less unchanged over the entire growth process and the mean of the fractal dimension is 1.716. The fractal dimension of the first stage is on the lower side because the viscous fingering has just set in and there is less complexity to the shape and thus has a lower fractal dimension. As the viscous fingering proceeds the fractal dimension is found to increase slowly and steadily reaching a value of 1.7409 for the final stage of growth where there is more of structural complexity associated with the pattern.

3. RESULTS AND DISCUSSION:

Viscous Fingering is studied in radial Hale Shaw cell using thick oil (HP90) more viscous fluid and air as less viscous fluid. Fractal analysis reveals details of structure and tex-

ISSN 2229-5518
ture associated with complex shapes that are difficult to analyze otherwise. It is observed that the fractal dimension and hence the complexity of shape associated with the patterns developed is more or less same with a mean fractal dimension of 1.716. In the initial stage of the development of the fingering pattern, the fingers are less complex in shape and thus exhibit a lower value of fractal dimension and hence a lower complexity of shape. As time increases, the fingering pattern grows further the fractal dimension is found to increase gradually and steadily. The fractal dimension of the viscous fingering pattern is found to be close to that of DLA patterns indicating similarity in the two processes giving rise to identical random pattern with their characteristic fractal dimension.

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